The Assessment of Stability and Hydrostatic Curves for Submarines at Surface and Submerged Conditions

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Abstract

In present paper, the stability curves for submarines at surface and submerged conditions have been presented in which the stability of ship has not been mentioned. The submarine due to the points such as diving and going under water are different from the ships. Three various conditions of stability in submarine have been considered which are as surface, transition and submerged conditions. The ballast tanks are provided for “submerging” process, so that the ballast tanks play important role in making the submarines stable and Equilibrium. The present paper is provided to give the results about the stability of submarine and it's difference points with the stability of ships.

Keywords: Hydrostatic curve, stability, submarine, surface, submerge, ballast tanks

Introduction

For several aspects, The stability and control characteristics of submarine and ship differ from each other in which it is assumed that the submarine’s hull is totally cylindrical, so that the main differences would be as following:

- The stability range of 180 degree: The stability range of submarine at surface and depth of water is 180 degree, whereas the stability range is less than 90 degree in the marine vessels. The maximum converter arm occurs at 90 degree angle. For this, \( GZ \) curve of the submerged submarine is close to a sine curve in the surface and depth of water, so that \( GZ = GM \sin \theta \) and \( GZ = BM \sin \theta \) at surface and depth of water, respectively. \(^1\),\(^2\)

- Surface Water Criteria (BG): Accordingly, the distance of \( B \) and \( G \) and their condition toward each other is not important in the static stability, but in the submarines in order to be assured of the stability condition, \( G \) has to be at the bottom of \( B \). In depth of water, \( M \) overlaps \( B \); In fact point \( B \) is the center point. At the surface of water while \( G \) is upper than \( B \), it means that in transition from the depth of water to the surface, \( G \) and \( B \) would be replaced with each other by
which $G$ has to be at the bottom of $B$. This situation is called the "the stability during the transition" which this situation is only possible for the submarine. Value of $BG$ in the submarine would be 3-5% of the pressure hull diameter. \[1\]

- The fixed situation of transverse center: In the ships, the transverse metacenter in big transverse rolling angles on a curve replaces to the shape of $\Box$ or $\square$, whereas in all transverse rolling angles and drafts in the submarines, even at the surface or depth of water, the metacenter is fixed and is located in the center of cylindrical hull, meaning that $KM_t=R$.\[3\]

- The necessity of longitudinal stability at depth of water: In basis of stability in ships, the transverse stability has been introduced as the main subject by which it is highlighted that the longitudinal stability is not that much important which this is due to the point that longitudinal metacenteric height is bigger than the transverse metacenteric height \[1\]. But in the submarines, we have $BG=GM_t=GM_l$ which it means that the metacenteric longitudinal and transverse height are equal in submarine. Due to the point that the possibility of replacing the longitudinal moments is much larger than the transverse moments, so that in the submarines the longitudinal stability is more important than the transverse stability.\[4\]

- Storage area of buoyancy: It is indicated that the storage area of buoyancy in the floating ship could be more than 50%, whereas the storage area of buoyancy is usually less than 15% in the submarines. \[1\] In the submarines with the floating monohulls, the storage of buoyancy could be up to 8%. \[5\] Storage of buoyancy in the submarines is at the very high volume of tanks, whereas this relation is not true in the ships.\[3\]

Two problems in keeping the submarines stable in capsizing situation have been presented as a change in tonnage, for instance the change in the value and situation of varied weight, a change in density of sea and etc, and the other changes in the form and value of buoyancy. To observe the situation, we use polygon trim by which the possibility of weight compensation and weight longitudinal moments for the submarine would be observed.

1The characteristics which are used in the stability observation are as following:

$D, V, T_{\text{Fore}}, T_{\text{Midship}}, T_{\text{Aft}}, h, \omega$ and $\theta$ are used which are respectively the indices of displacement, the volume of displacement, fore Draft, midship draft, aft draft, the height of transverse metacenter, trim angle, list angle. To observe the stability at surface condition, four various conditions could be observed:

- Stability at small trim and list angles
- Stability at large trim angles
- Stability at large list angles
- Ascent and decline

Submarine at small list and trim angles would be observed in such a way that draft has not to be the reason for intermitting the pressure hull through the upper edge. This situation would be only possible within the operational cases while the fuel gets to be ended. For this, the list and trim angles have to be verified at $10^\circ \geq \theta$ and $1.5^\circ \geq \Box$. In these cases, the stability and buoyancy of submarine have to be obtained through the primary hydrostatic curves. The issues related to the surface stability in the ships and also the ones which are applied for the submarines at the surface of water would not be discussed in present paper. For this, we could refer to the hydrostatic curves ($TPC, MCTC, WPA$), Bonjan, $KN$ and the cruciate trim `s diagram. In present paper, we have discussed about the curves such as the curve for the geometric characteristics of ballast tanks and $V=f(T)$, domical curves, longitudinal static stability, transverse metacenter in the slope toward the fore and the application of these curves is mentioned in this paper.
Main Body
The primary hydrostatic curves
In these curves, in terms of mean draft and under the condition “even keel” in list and trim angles, the following cases have to be drawn:

\[ V, LCB, VCB, KM_t, BMI \text{ and } LCF \]

have to be drawn which are respectively the volume of displacement, the distance of buoyancy from the mid edge, the height of buoyancy center from the baseline, the height of metacenter from the baseline, the longitudinal metacenteric radius, and the distance of draft chip from the mid section. To determine the submarine characteristics through the curves, we have only to have the displacement volume or the submarine’s mid draft, for instance a sample of these curves have been shown in figure A.1.

Figure A.1: The curve for hydrostatic characteristics

To determine the hydrostatic characteristics of submarine in particular conditions like at surface or at small heel and trim angles, we have to act as following:

- Determine the condition of mass center
- Determine the volume of displacement based on the relation between volume, density, and based on the volume we have to determine the mid draft, \( LCB, KM_t, LCF \) and \( BMI \).
- Finding the Tangent of trim angle through Eq. (A.1)

\[ \tan \psi = \frac{(LCB-LCG)}{BMI} \quad \text{Eq. (A.1)} \]

- Calculating the drafts through the following equations

\[ T_{\text{fore}} = T_{\text{midship}} - L_{\text{fore}} \times \tan \psi \quad \text{Eq. (A.2)} \]
\[ T_{\text{aft}} = T_{\text{midship}} + L_{\text{aft}} \times \tan \psi \quad \text{Eq. (A.3)} \]

Where \( T_{\text{fore}} = T_{\text{aft}} \)

- Determine the value of metacentric transverse height beside the effect of tanks using the following equation:

\[ GM_t = KM_t - KG - FSC \quad \text{Eq. (A.4)} \]
1.2. The Stability of Shallow Submarine at Large Trim Angles

In this part, the conditions of shallow submarine at large trim angles ($\theta \geq 1.5^\circ$) would be observed. In the other word, the minimum angle appears at the time while the water line intermits the upper edge of pressure hull. This condition may occur at the operative situations. For instance, in this relation we could refer to the Curves of main ballast tanks. To observe the hydrostatic characteristics of submarine at large trim angles, several diagrams would be needed.\cite{3}

1.2.1. Curves of Lines in Main Ballast Tanks

Inundation rate would be considered in the curves which are presented for each of the ballast tanks. We could mention the curves as following:

In this case we have $V_n, M_x, M_z, M_c$ and $I_{xx}$ which are respectively the volume of tanks, the longitudinal moment of mid section ($m^4$) and the transverse moment of central line ($m^4$). The surface of the water in the tanks is considered from the baseline, and it is shown in terms of $T$, and also it is considered as a vertical axis diagram. The start point of curves is the residual water surface in the depth of tanks. We could obtain the values of the characteristics which are related to the reservoirs which this is possible with the surface water of the tanks. In following, the data for each of the marine’s ballast tanks have been represented in figure A. 2.

Figure A. 2: A sample of curve presented for the geometric characteristics of ballast tanks.

![Figure A. 2: A sample of curve presented for the geometric characteristics of ballast tanks.](image)

1.2.2. $V=f(T)$ Curves

These curves are designed to determine how much water went through the main ballast tanks, which also they are represented to increase the draft in surrounding the tanks, for this the volume of the water which went through the ballast tank would be high which it is computed through the draft value. Origin of coordinate in each of the curves, the symmetric point of main axis in each tank and the draft line in the tonnage condition are all normal. In the other word, it could be said that these kinds of curves are similar to the Bonjan curve. To determine the amount of water which went through the ballast tanks with changing the condition of submarine, we have to act as following:

- Draw the draft line on the curve through knowing the amount of fore-and-aft and mid drafts. The symmetric point of the draft point and each of the main vertical axises would give us the draft in surrounding the ballast tank.
- We draw the horizontal lines from the symmetric points in which the $V=f(T)$ Curves would be fractured.
• Through measuring the length of the horizontal lines, the scaled volumes have been obtained which are equal to the amount of water which went through the ballast tanks.
• To determine the transverse, longitudinal and vertical moments, we could use the curve with the geometric characteristics in the ballast tanks in which the water went through the tanks. In figure A.3, this figure has been drawn which shows all the specialties.

1.2.3. Buoyancy Diagram at Fore-and –aft Inclinations
This diagram represents the following curves:
• The same slope at fore-and –aft inclinations \( \alpha = f_1(V, MX) \)
• The same draft at mid ship \( T_{midship} = f_2(V, MX) \)

Here we have \( V \) and \( MX \) which are the volume of displacement and the longitudinal moment of displacement volume (\( \text{m}^4 \)), respectively. In the prototype submarine, the curves at fore-and –aft inclinations for -12 to 12 degree with one degree distances, and the fixed draft curves for 5-7.5 meter drafts have been drawn. The fixed slope curves show the change in moment \( (M_x = V \cdot X) \) while the displacement increases due to the Gradual dive in the submarine. Draft at mid ship for all the fixed slope curves has been computed by which the fixed draft curves have been obtained. \([3]\)

To use the diagram, we have to act as following:
• The volume of floating displacement has to be specified on the vertical curve by which a horizontal line would be drawn.
• The longitudinal moment of the displacement volume from the mid section has to be specified on the horizontal axis by which a vertical line would be also drawn.
• The symmetric point of direct lines show the Equilibrium mode within the assumed tonnage for the submarine
• The values of mid Draft and trim angle for this point could be obtained through the Graphical interpolation

1.2.4. Longitudinal Static Stability Diagram
To determine the longitudinal stability, we could use the domical diagram by which the longitudinal stability curve could be resulted. To determine the longitudinal stability diagram and the value of trim angle, we could act as following:
• Use the domical diagram and gain the values about the MX for all the negative and positive angles at a horizontal symmetry with the fixed slope curves.
• Specify the trim angle’s values at the horizontal axis in the diagram and draw a vertical axis from the obtained values.
• From each vertical lines, separate MX-MX value and connect them to each other.

\[ M_{right} \psi = M_X - M_{X \psi} = f(\psi, V=\text{const}) \]  \hspace{1cm} \text{Eq.(A.5)}

• The value of trim angle for the submarine at Equilibrium mode would be obtained from the symmetric point of the curve with the \( \psi \) axis. \( M_{right} \psi \) is well known with the name "longitudinal static stability diagram ". Through this diagram moreover the trim angle for \( \psi_{max} \) and \( L_{max} \), the maximum value of longitudinal converter moment could be determined. The value of \( L_{max} = M_{right} \psi / V \) in terms of meter is the criterion for submarine’s longitudinal static stability at the tonnage condition, which it is known with the maximum arm of the longitudinal static stability.

In fig A.4, the curves of longitudinal convertor moments for the following rolling conditions have been presented:
• At surface condition
• Trim condition for diving
• Frequent ascent and diving at one stage
• Frequent ascent and diving at two stages
Figure A.4: Longitudinal Static Stability Diagram

In this diagram, the vertical axis of longitudinal convertor moment $M_{right\psi}$ (m$^4$) and the horizontal axis of trim angle ($\psi$) is in terms of degree. The curves have been drawn at 30 degree. The curves of longitudinal static stability are used to determine the following factors:

- The value of the static or dynamic trim angles for a certain trim moment
- The value of trim which is relevant with the trim angle

1.2.5. $KMt$ Transverse Metacentric Curves at fore-and –aft Inclinations

This diagram represents the curves $KMt = f(V, \psi = const)$ in which the changes in $KMt$ values during the submarine diving is fixed at trim angel. Curves are drawn separately for the negative and positive angels (0-12 degree). The vertical axis for $KMt$ values is in terms of meter and the horizontal axis of displacement volume values is in terms of m$^3$. For instance, observe the curves for a fore- and –aft inclinations in fig A.5. [3]

Figure A.5: KMT Transverse metacentric curves at aft inclinations
For the submarine which is at Equilibrium mode, the values of $K_{Mt}$ through these diagrams and the graphical interpolation method would be obtained through the displacement volume and trim angle.

### 1.2.6. Stability in Submarine at Large Trim Angles

Metacentre has a direct relationship with a ship's rolling period. A ship with a small $GM$ have a long roll period. An excessively low or negative $GM$ increases the risk of a ship capsizing in rough weather. It also puts the vessel at risk of potential for large angles of heel if the cargo or ballast shifts. A ship with low $GM$ is less safe if damaged and partially flooded because the lower metacentric height leaves less safety margin. A larger metacentric height on the other hand can cause a vessel to be too "stiff"; excessive stability is uncomfortable for passengers and crew. This is because the stiff vessel quickly responds to the sea as it attempts to assume the slope of the wave. An overly stiff vessel rolls with a short period and high amplitude which results in high angular acceleration. This increases the risk of damage to the ship and to cargo and may cause excessive roll in special circumstances. Roll damping by bilge keels of sufficient size will reduce the hazard. Criteria for this dynamic stability effect remains to be developed. In contrast a "tender" ship lags behind the motion of the waves and tends to roll at lesser amplitudes.

### Discussion and Results

Solving the problems about the buoyancy and stability in submarine at large trim angels

The buoyancy and stability in submarine at large trim angles are determined through the approximations, domical diagram, and transverse metacentric curves at fore-and –aft inclinations. To solve the problems, we have to act as following:

- Provide a primary approximation of tonnage after the changes like sealing several ballast tanks.
- Use the curves “primary hydrostatic characteristics “ and determine the submarine characteristics based on the loading base, the main factors are included of draft and trim angle.
- If the trim angle be bigger than 1.5 degree, in this case we have to use the domical diagram and more approximations would be needed by which other characteristics have to be specified as well.
- In base of primary approximations, the amount of water in each ballast tanks for the sealing has to be specified with the $V=f(T)$, in this case the second approximation of taking on a cargo is needed.
- In base of second approximation and domical diagram, determine the buoyancy characteristics. Through the graphical interpolation method, determine the $T_{midship}$ and draft at mid section $T_{midship}$.
- In base of second approximation, specify the water which went through the ballast tanks and present the third approximation. Take into attention that the approximations would be provided up to the time while the difference between two approximations is trivial.
- While the final approximation was provided, through the curve of geometric characteristics for main ballast tanks, the vertical moment from the baseline and the transverse moment from the central line have to be specified. Through the transverse metacentric curves at fore-and –aft inclinations, the height of transverse metacenter from the baseline ($K_{Mt}$) has to be specified. If the moment let the list angle in submarine be larger than 10 degree, so that in order to determine the characteristics of transverse stability, we have to use the curves of transverse static stability. These curves are the very $GZ$ curves which have been multiplied at tonnage.

The curves of transverse static stability

- Surface conditions
- Trim conditions for diving
- Frequent decline and ascent at one stage
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- Frequent decline and ascent at two stages
- Capsizing conditions

In this diagram, the vertical axis and the horizontal axis are respectively the transverse convertor moment \( M_{\text{right}} \) (\( m^4 \)) and the list angel(degree). For this, Observe fig. A.6.

**Figure A.6: The curve of transverse static stability**

In this case, the curves are used to determine the following cases:
- Determine the value of list or heel angel for the certain list moment
- The value of list moment to obtain the list angle
- The coefficient of transverse stability and the height of transverse metacenter
- The margin of transverse stability

To determine the list and trim angel, we have to specify the list moment on the vertical axis by which a horizontal line has to drawn. In this case, we could obtain the symmetric point of horizontal line and the curves for the list angle. To determine the list angle, the area between the vertical line and \( S_{\text{incl}} \) curve have to be equal with each other. If the vertical line be drawn at the right side of the area, then the symmetric point of the vertical line and horizontal axis of diagram for the list angle would be obtained. To obtain the value of list moment from the list angel, we act vise versa of the process in which the list angel is determined. To determine the coefficient of transverse stability, we have to find a point on the curve which is proportional to the list angel. From this point, draw a horizontal line and tangent on the curve. In consistency with the angel 1, separate the 57.3 degree radian on the horizontal line. From the end of this line, draw a vertical line to intermit the tangent line. The length of the vertical line is equal to \( V.GM_t \).

The height of transverse metacenter is determined through the following equation:

\[
H = \frac{V.GM_t}{V}
\]

**Eq.(A.6)**

The margin of transverse stability realizes with the maximum list moment in which the permissible angel would be determined in the submarine. Permissible angel interrelates with the angel of list admissible which this is provided for the persistent performance of machineries, and the angel of heel admissible is needed for the temporary performance of the machineries. The margin of transverse static stability within the difference between the value of \( M_{\text{right}} \) and the direct horizontal line in consistent with the heel moment \( M_{\text{incl}} \) are equal with each other.

The frequent ascent and decline

The frequent ascent and decline in submarine is possible only at stage one or two. In most cases, the main ballast tanks in submarine are divided into mid and last categories. With this assumption, the decline and ascent schematics would be as following. While the ascent in submarine is possible in stage one, in this case at normal situation in submarine, seal all the mid and last ballast tanks with each other. While the ascent in submarine is only possible at stage two, in this case in
submarine, the last ballast tank and the mid ballast tank has to be sealed at stage one and two, respectively.

Essential factors in observing the stability and buoyancy of submarine at decline and ascent conditions are as following:

Where \( V \), \( T_{mean} \) and \( h \) are respectively the volume of displacement, the mid draft from the BL point, the transverse metacentric height.

The curves of primary transverse stability at frequent ascent and decline cases

These curves are used to determine the transverse metacentric height.[4]

In this diagram in terms of mid draft from the BL line, the following curves have been drawn. Where \( V \) and \( KM_{ts} \) are displacement and transverse metacentric height from BL, through the water surface in the ballast tanks and the floats in the tanks in the pressure hull, the transverse metacentric height from BL would be possible at this time. Also, the equations as following have to be provided which the first and second equations are provided for one and second stage:[4]

\[
KM_{f1} = KM_{ts} - FSC \quad \text{Eq.(A.7)}
\]

\[
KM_{f2} = KM_{ts} - FSC \quad \text{Eq.(A.8)}
\]

In this case, the height has to be from (BG) to the gravity center (CG), the \( KG_1 \) and \( KG_2 \) have to be provided for one and second stage.

The followings have to be verified in order to draw the curves:

- Ascent and decline in submarine has been observed without any movement
- Trim and list angles in the Ascent and decline process is zero
- Each of the main ballast tanks in terms of volume percentage has been sealed
- At any moment of frequent Ascent and decline in submarine, through the curves we could obtain the primary metacentric height from the difference points between the values of \( KM_t \) and \( KG \). For instance, the curves for a submarine have been represented.[5]

**Figure A.7:** The primary transverse stability curves at frequent ascent and decline
Conclusion
We could mention the ballast tanks as the most important reason in difference between the stability of ship and submarines. As a matter of fact, in submarines ballast tanks are used to allow the vessel to submerge, water being taken in to alter the vessel's buoyancy and allow the submarine to dive. Ships designed for carrying large amounts of cargo must take on ballast water for proper stability when travelling with light loads and discharge water when heavily laden with cargo. A submarine may have several types of ballast tank: the main ballast tanks, which are the main tanks used for diving and surfacing, and trimming tanks, which are used to adjust the submarine's attitude both on the surface and when underwater.

References